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A utility-based bicycle speed choice model with time and energy factors

Alexander Bigazzi[1] · **Robin Lindsey**[2]

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Abstract
This paper presents a utility-based behavioral model of bicycle speed choice. A mathematical framework is developed with travel time, energy expenditure, and control factors. Observational speed data are used to calibrate the model and estimate marginal rates of substitution between energy expenditure and travel time. The model is validated by applying it to predict speed changes on pedal-assist electric bicycles. This paper lays a foundation for further development of operational active travel speed and joint speed-route choice models, which can lead to more sensitive and behaviorally-grounded operations, microsimulation, and mode choice models. In addition, the findings have implications for modeling the effects of emerging bicycle technologies. Further research is needed to calibrate the model for a broad population of travelers.

Keywords Bicycles · Electric bicycles · Energy expenditure · Speed choice · Safety · Utility maximization

Introduction

Many cities have implemented programs and policies to increase active travel (primarily walking and bicycling) as means of urban transportation. Correspondingly, transportation professionals have growing needs for analysis techniques to forecast an array of active travel behaviors, including mode, route, and speed choices. Most existing analysis tools address mode choice, but many treat travel speed as constant and assume that bicyclists and pedestrians choose shortest-distance routes. Better operational-level models of intra-modal active travel behavior are needed for a variety of purposes: predicting traffic volumes for infrastructure and service planning, assessing the impacts of route quality on mode and

✉ Alexander Bigazzi
alex.bigazzi@ubc.ca

Robin Lindsey
robin.lindsey@sauder.ubc.ca

[1] Department of Civil Engineering, School of Community and Regional Planning, University of British Columbia, 2029 – 6250 Applied Science Lane, Vancouver, BC V6T 1Z4, Canada

[2] Sauder School of Business, University of British Columbia, 2053 Main Mall, Vancouver, BC V6T 1Z2, Canada

destination choices, quantifying health benefits and risks of active travel, and assessing active travel as a means to enliven public spaces and spur commercial activity, among other applications (Ishaque and Noland 2008).

Utility-based models of bicycle routing behavior have recently been developed that include the influences of route attributes such as road grade, facility type, and trip type (Broach et al. 2012; Hood et al. 2011; Sener et al. 2009). These models neglect endogenous speed choice, and combine the disutility of additional travel time due, for example, to hill ascents and road crossings with the utility effects of other factors such as energy expenditure, comfort, and safety. Conflating these effects into a single parameter is behaviorally ambiguous. It can also bias forecasts, particularly if the relationships among these factors change due to new intersection treatments or the proliferation of electric bicycles, for example. Modeling joint speed and route choices could lead to more accurate, sensitive, and useful analysis tools. Bicycle speed choice is also important for roadway operations, developing microsimulation models (Twaddle et al. 2014), and estimating and ameliorating crash risks and health impacts.

For motorized modes, it has generally been assumed that travel speed is chosen to minimize travel costs. When travel time is assumed to be the only cost this implies that the driver chooses the highest feasible speed consistent with speed limits and road conditions. A more comprehensive approach includes vehicle operating costs, which increase with speed above a certain level. (Mohring 1965) used the trade-off between travel time and vehicle operating costs to estimate the value of travel time. Later studies accounted for additional factors such as safety and the risk of traffic citations (MacFarland and Chui 1987). Speed choice modeling was also used in the development of a behavioral traffic congestion model in a series of papers in the early 2000's with the goal of "endogenizing speed choice" (Verhoef and Rouwendal 2004). These motorized speed choice models assume that observed speeds correspond to an equilibrium in which motorists are minimizing some private travel cost function.

For active travelers, speed choice is more complex because it can involve non-negligible trade-offs among travel time, energy expenditure, safety, and stability, among other potential factors. A number of papers have measured walking and bicycling speeds in various conditions (Hediyeh et al. 2014; Landis et al. 2004; Minetti et al. 2003; Parkin and Rotheram 2010), and several empirical regression models have been developed to identify the external factors correlated with active travel speed, such as road grade, facility type, trip type, and traveler gender (El-Geneidy et al. 2007; Silva et al. 2014; Strauss et al. 2016). These studies are informative, but similar to the route choice models they conflate the effects of individual factors that can be expected to influence speed choice, such as time and energy expenditure. Other research has examined aggregate flow-constrained bicycle traffic characteristics as analogous to motorized traffic flow theory (Jiang et al. 2016; Jin et al. 2015; Navin 1994; Zhang et al. 2013). Ishaque and Noland (2008) propose a theoretical framework in which a pedestrian's speed choice is determined by their capabilities, their value of time, and the potential risks they take. The authors suggest that pedestrians make trade-offs among these factors when choosing a travel speed, but do not specify the model beyond this general identification. To our knowledge, no studies have gone beyond this in specifying a behavioral model of active travel speed choice.

This paper frames and formulates a utility-based bicycle speed choice model for free-flow conditions. A mathematical framework is presented, and trade-offs among energy expenditure, travel time, and stability/risk are identified and discussed. Observational data from past research are used to estimate marginal rates of substitution between energy expenditure and travel time. The model is validated by applying it to speed changes on electric bicycles. This paper lays the foundation for further development of operational active travel speed and joint speed-route choice models. In addition, by quantifying the rate at which bicyclists trade off time and energy, the findings have implications for bicycle network planning and for modeling the effects of emerging bicycle technologies.

An analytical speed choice model

This section formulates a model in which a traveler chooses bicycle speed to maximize expected utility by considering trade-offs among several factors, similar to the generalized cost minimization models of (MacFarland and Chui 1987; Verhoef and Rouwendal 2004) and the formulation of (Ishaque and Noland 2008). Building on those models, utility U from travel over a fixed (unit) distance is specified as a function of travel time t, energy expenditure rate e, and bicycle control c (all of which depend on speed v), as well as the static positive utility derived from making the trip by bicycle θ (which is assumed to be sufficiently large to make the trip worthwhile):

$$U = f(t(v), e(v), c(v), \theta) \tag{1}$$

Variable c reflects the rider's ability to control the bicycle and avoid a crash or fall. This depends on various factors including stability (related to balance and steering) and collision avoidance (related to perception-reaction time, braking distance, and potential crash severity). In general, the traveler`s choice of speed will vary with the road grade, traffic signals, wind and other factors. Changes in speed are subject to short time lags because of the mass of the bicycle and rider. The dynamics of speed adjustment are not considered here, but desired/target speed is an important input for modeling acceleration and deceleration events (Ma and Luo 2016; Twaddle and Grigoropoulos 2016).

The rider's preferred speed is derived by taking the total derivative of U with respect to speed v

$$\frac{dU}{dv} = MU_t \frac{dt}{dv} + MU_e \frac{de}{dv} + MU_c \frac{dc}{dv} \tag{2}$$

where $MU_t = \frac{\partial U}{\partial t}$, $MU_e = \frac{\partial U}{\partial e}$, and $MU_c = \frac{\partial U}{\partial c}$ are the marginal utilities of travel time, energy expenditure, and control, respectively. At a steady-state equilibrium, utility maximization implies a preferred speed, v^*, i.e., the "cruising speed", unimpeded by other vehicles and travelers. If $\frac{dU}{dv}$ is continuous and non-increasing with v for $v > 0$ (i.e., U is concave), there is a single utility-maximizing speed v^* that satisfies the first-order condition $\frac{dU}{dv} = 0$.[1]

The trade-off between travel time and energy expenditure can be measured with the marginal rate of substitution,

$$MRS_{et} = \frac{MU_e}{MU_t}. \tag{3}$$

At equilibrium speed v^*, $\frac{dU}{dv} = 0$ and MRS_{et} can be calculated by rearranging Eq. (2):

$$MRS_{et} = -\frac{\frac{dt}{dv}}{\frac{de}{dv}} - \frac{MU_c}{MU_t} \frac{\frac{dc}{dv}}{\frac{de}{dv}} \tag{4}$$

[1] A cost-minimizing alternative formulation could similarly be derived; the utility approach is used here because none of the factors that determine speed is measured in monetary units.

In the next three subsections we discuss each term on the right-hand side of Eq. (2).

Travel time

The relationship between steady-state speed (in m/s) and per-km travel time (in min/km) is $t = \frac{1000}{60}\frac{1}{v}$, leading to

$$\frac{dt}{dv} = \frac{-1}{0.06v^2}.$$

The marginal utility of travel time for a given trip is assumed to be a constant, $MU_t < 0$, consistent with (Börjesson and Eliasson 2012) for trips up to 40 min.[2] Consequently, $MU_t\frac{dt}{dv}$ is assumed to be positive, differentiable, and decreasing with v for $v > 0$.

Energy expenditure

Following (Glass and Dwyer 2007) the steady-state rate of energy expenditure while bicycling, e, in kcal/min is modeled as a linear function of the tractive power transferred to the bicycle, p, in watts (W):

$$e = \delta_0 + \delta_1 p.$$

The intercept term, δ_0, is positive because a bicycle rider has some base level of energy expenditure even while coasting. The power required to maintain a steady bicycle speed can be computed from two parameters representing rolling resistance and road grade $\left(\mu_1\right)$ and aerodynamic resistance $\left(\mu_3\right)$,

$$p = \max\left(0, \mu_1 v + \mu_3 v^3\right).$$

(The second coefficient is called μ_3 rather than μ_2 to highlight that the second term varies with speed cubed rather than speed squared). Parameters μ_1 and μ_3 in turn depend on roadway and traveler characteristics. Parameter μ_1 is specified by the function

$$\mu_1 = \left(m + m_b\right)g\left(G + C_R\right)$$

where μ_1 is measured in W s/m, rider mass m in kg, bicycle mass m_b in kg, gravitational acceleration $g = 9.8$ m/s^2, road grade G is unitless, and coefficient of rolling resistance C_R is unitless. Parameter μ_3 is specified by the function

$$\mu_3 = 0.5\rho C_D A_F$$

where μ_3 is measured in W s^3/m^3, air density $\rho = 1.23$ kg/m^3, drag coefficient C_D is unitless, and frontal area A_F is measured in m^2 (Bigazzi and Figliozzi 2015; Martin et al. 1998; Olds 2001; Wilson 2004). If the bicyclist is coasting or braking on a negative grade sufficiently steep to overcome resistance forces, then $v^2 \le \frac{-\mu_1}{\mu_3}$ and $e = \delta_0$. Otherwise, $p > 0$ and $e = \delta_0 + \delta_1\left(\mu_1 v + \mu_3 v^3\right)$, and the bicyclist must expend more energy to maintain higher

[2] The value of travel time is highly heterogeneous and context-dependent (Small 2012), but this model only requires MU_t to be independent of trip duration, not constant across modes or persons. There is some evidence of positive utility of travel time in certain situations, particularly during active travel (Mokhtarian et al. 2015). However, for utilitarian bicycle trips the marginal utility of travel time is expected to be negative (Börjesson and Eliasson 2012).

speeds at a given road grade. Parameters δ_0, δ_1, and μ_3 are all expected to be positive, as described below; the sign of μ_1 depends on the road grade G. This formulation of $e(v)$ assumes a bicycle with sufficient gearing, and a rider with sufficient skill, to achieve any desired power/speed combination at a moderate cadence.

From this formulation of $e(v)$,

$$\frac{de}{dv} = \begin{cases} \delta_1\left(\mu_1 + 3\mu_3 v^2\right), & v^2 > \frac{-\mu_1}{\mu_3} \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

If $\mu_1 \geq 0$, Eq. (5) simplifies to $\frac{de}{dv} = \delta_1\left(\mu_1 + 3\mu_3 v^2\right)$; otherwise, there is a discontinuity at $v = \sqrt{\frac{-\mu_1}{\mu_3}}$ where $\frac{de}{dv}$ jumps from 0 to $-2\delta_1\mu_1$ (a positive number). Hence, $\frac{de}{dv}$ is non-negative and non-decreasing with v.

MU_e is the traveler's marginal utility of energy expenditure, which can be positive or negative. It depends on comfort or discomfort experienced during and after exercise of increasing intensity, as well as expected future benefits from improved cardiovascular health and physical appearance. We are not aware of any studies of MU_e during active travel. Large inter- and intra-person variability can be expected, depending on a traveler's physical condition, recent physical activity, weather, clothing, and other factors. MU_e is likely to be higher for active travelers according to some attitudinal research (Gatersleben and Appleton 2007; Heinen et al. 2010).

We assume that, for a given traveler, MU_e is a non-increasing function of e, and negative at very high values of e reflecting discomfort from extremely high workloads. If MU_e is positive at low values of e, there exists an ideal energy expenditure level, e^*, at which $MU_e = 0$. Similar to the treatment of MU_t, we assume that MU_e is constant throughout a trip, and hence independent of trip duration. In practice, it could decrease as a traveler tires or becomes more uncomfortable in very hot or very cold weather.

In summary, $MU_e\frac{de}{dv}$ can be positive or negative, and if $\mu_1 < 0$ it has a discontinuity at $v = \sqrt{\frac{-\mu_1}{\mu_3}}$. If $MU_e < 0$, $MU_e\frac{de}{dv}$ is non-positive and non-increasing with v.

Control

There has been little research on either the effect of speed on control $\left(\frac{dc}{dv}\right)$ or the marginal utility of control $\left(MU_c\right)$. Consequently, it is more difficult to formulate the influence of control on speed choice than the influence of travel time and energy. Given our focus on the time/energy trade-off, we make several general assumptions about the product $MU_c\frac{dc}{dv}$ for tractability, and leave a more detailed exploration for future work. Note that even if an objective function relating speed to crash risk exists, it may not accurately describe individuals' perceived risks which determine their preferred speeds. Indeed, bicyclists may derive utility from the sensation of motion or speed on a bicycle (Gatersleben and Appleton 2007; Mokhtarian et al. 2015). Such a preference is not explicitly modeled here although it could be included as another variable in the utility function.

In the model, speed is influenced by control via the term $MU_c\frac{dc}{dv}$, which represents the marginal change in control-related utility with speed. Control-related utility is expected to be low at very slow speeds due to the difficulty of maintaining balance, maintaining a straight course, shifting gears, and avoiding coming to a dead stop on a steep hill. Control-related utility is also low at high speeds due to short crash avoidance times, high potential crash severity, loss of traction on slippery or loose surfaces, and potential loss of contact with the ground on rough surfaces. In light of these observations we define three regions of speed for subsequent analysis:

I. Low-speed instability, where $MU_c \frac{dc}{dv} > 0$ (higher speed yields more control and related utility),
II. Mid-speed stability, where $MU_c \frac{dc}{dv} \approx 0$ (control and related utility are not strongly influenced by speed),
III. High-speed crash risk, where $MU_c \frac{dc}{dv} < 0$ (higher speed increases crash risk and decreases control-related utility).

These regions define an inverted "U"-shaped function relating speed to control. For convenience we assume that $MU_c \frac{dc}{dv}$ is continuous and non-increasing for $v > 0$.

Equilibrium conditions

From the preceding formulation, $\frac{dU}{dv}$ is defined over $v > 0$, but has a discontinuity at $v = \sqrt{\frac{-\mu_1}{\mu_3}}$ if $\mu_1 < 0$. Thus, the maximum utility occurs at a speed v^* where either $\frac{dU}{dv} = 0$ or $v^* = \sqrt{\frac{-\mu_1}{\mu_3}}$. The time and control terms of Eq. (2) are decreasing over $v > 0$, and the energy term is non-increasing when $MU_e < 0$. Hence, when $MU_e < 0$ any solution of $\frac{dU}{dv} = 0$ is unique and constitutes a local utility maximum.

At an observed steady-state cruising speed where the bicyclist is pedaling and unimpeded by other vehicles and travelers, $p > 0$ and $v^{*2} > \frac{-\mu_1}{\mu_3}$, and MRS_{et} can be computed from substitutions into Eq. (4):

$$MRS_{et} = \frac{1}{\delta_1\left(\mu_1 + 3\mu_3 v^{*2}\right)}\left(\frac{1}{0.06v^{*2}} - \frac{MU_c}{MU_t}\frac{dc}{dv}\right) \tag{6}$$

If v^* is a moderate speed in region II of the control term, then $MU_c \frac{dc}{dv} \ll \left(MU_t \frac{dt}{dv} + MU_e \frac{de}{dv}\right)$ and Eq. (6) can be approximated by

$$MRS_{et} = \frac{1}{0.06v^{*2}\delta_1\left(\mu_1 + 3\mu_3 v^{*2}\right)} \tag{7}$$

The effect of the control term is to moderate v^*: a higher v^* at low speeds (region I) and a lower v^* at high speeds (region III). In the low-speed region I where $MU_c \frac{dc}{dv} > 0$, Eq. (7) would *underestimate* the true MRS_{et}; in the high-speed region III where $MU_c \frac{dc}{dv} < 0$, Eq. (7) would *overestimate* the true MRS_{et}.

For region II of the control term, the equilibrium speed must be such that $e > e^*$ and $MU_e < 0$ because $MU_t \frac{dt}{dv}$ and $\frac{de}{dv}$ are both positive. An effort level satisfying $e \leq e^*$ can only be observed at an equilibrium speed in region III where $MU_c \frac{dc}{dv} < 0$ (i.e., during steep descents) – in which case Eq. (7) would not be valid. In region I or II, $e > e^*$ necessarily. Nevertheless, this does not preclude $e^* > 0$.

If control effects are ignored, the preferred speed with $p > 0$ can be calculated from traveler and roadway attributes (δ_1, μ_1, and μ_3) and an estimated MRS_{et} by rearranging Eq. (7),

$$3\mu_3 v^{*4} + \mu_1 v^{*2} - \frac{1}{0.06\delta_1 MRS_{et}} = 0,$$

and solving the quadratic equation in v^{*2} for the positive root:

$$v^* = \sqrt{\frac{1}{6\mu_3}\left(\sqrt{\mu_1^2 + \frac{200\mu_3}{\delta_1 MRS_{et}}} - \mu_1\right)} \tag{8}$$

Because the moderating influence of control is neglected, Eq. (8) understates the true value of v^* at low speeds (region I) and overstates it at high speeds (region III). Since Eq. (8) is derived for the case $p > 0$, it only applies if $\mu_1 > -\sqrt{\frac{\mu_3}{0.12\delta_1 MRS_{et}}}$ or in terms of road grade

$$G > \frac{-1}{(m + m_b)g}\sqrt{\frac{\mu_3}{0.12\delta_1 MRS_{et}}} - C_R. \tag{9}$$

Model calibration and estimation of MRS_{et}

To calibrate the speed choice model and investigate how MRS_{et} (and by implication MU_e) might vary with e, we use speed observations at varying road grades from two previous studies of conventional bicycles (without power assistance). Parkin and Rotheram (2010) report average speeds at varying road grades for 16 bicyclists, along with parameter estimates $A_F = 0.616$ m^2, $C_D = 1.2$, $C_R = 0.008$, $(m + m_b) = 95$ kg, and $\rho = 1.226$ kg/m^3. A second source for v^* observations is the GPS data used in a study of bicyclist ventilation in Portland, Oregon (Bigazzi and Figliozzi 2015). The data set includes 55 h of 5-s aggregated speed, acceleration, and road grade data for three bicyclists. For each participant (A, B, and C), we calculate time-mean speed in 1% road grade bins with at least 30 observations. In an attempt to capture equilibrium cruising speeds, the calculation is restricted to speed observations greater than 1 kph (0.28 m/s) with absolute acceleration less than 0.1 kph/s (0.028 m/s^2) and road grade changes of less than 1%. Estimates of A_F, C_D, C_R, m, m_b, and ρ for each participant are provided in Bigazzi and Figliozzi (2015). The two data sets yield similar parameter values except for C_R which is half as large, 0.004, in Bigazzi and Figliozzi (2015).

Table 1 lists v^* at varying road grades for the two data sets. Observed average speeds vary among the riders and data sets, as expected, due to differences in mass, bicycle resistance parameters, marginal utilities of time and energy expenditure, and potentially other factors not included in the model. Both data sets (and casual observation) confirm that speeds tend to decline with increasing road grade, which indicates that $MU_e < 0$ at observed equilibrium speeds. Energy expenditure at v^* increases with road grade in both data sets.

From American College of Sports Medicine equations for energy expenditure during cycling (Glass and Dwyer 2007), we use $\delta_0 = 0.035m$ kcal/min and $\delta_1 = 0.058$ kcal/min/W,

Table 1 Observed average speeds at varying road grades (m/s)

Road grade (%)	Portland A	Portland B	Portland C	Parkin
− 3	6.4	[a]	[a]	6.7
− 2	6.3	[a]	[a]	6.5
− 1	5.6	5.0	5.2	6.3
0	5.4	4.7	4.7	6.0
1	5.0	4.5	4.2	5.6
2	4.4	4.0	3.8	5.2
3	4.1	[a]	[a]	4.8
4	3.6	[a]	[a]	4.4
5	3.4	[a]	[a]	4.0
6	3.1	[a]	[a]	3.6

[a] Fewer than 30 cruising speed observations (more data were collected for participant A)

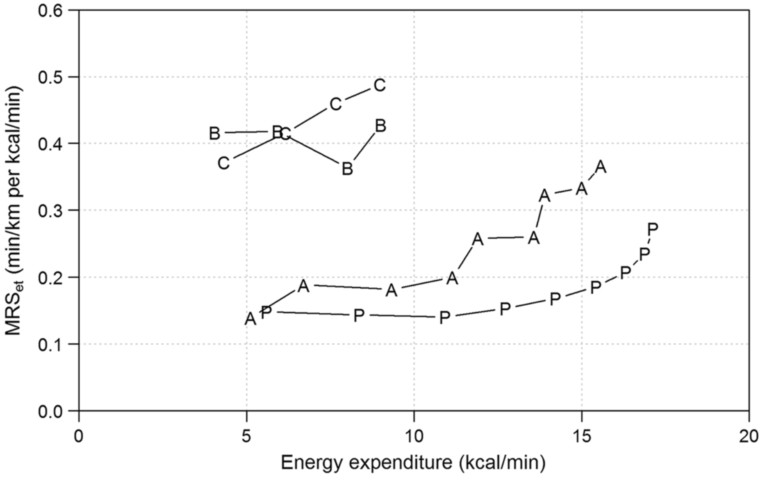

Fig. 1 Energy-time trade-offs from Portland (A, B, C) and Parkin (P) data sets

which includes a metabolic unit conversion of 5 kcal = 1 LO_2 (liters of oxygen) and 95% efficiency in the bicycle drivetrain (from pedals to tractive force, which accounts for losses in the chain and frame) as used in Parkin and Rotheram (2010).

Figure 1 gives MRS_{et} (Eq. 7) for the speeds in Table 1 at grades of − 2 to + 6% in the Parkin data ("P") and the Portland data (participants "A", "B", and "C"). Grades of − 3% and lower suggest braking (negative power) in both data sets, with $v^* < \sqrt{\frac{-\mu_1}{\mu_3}}$. MRS_{et} measures energy-time trade-offs in min/km per kcal/min, based on the units of t and e. Road grade increases and v^* decreases with higher e toward the right side of Fig. 1. Higher values of MRS_{et} suggest increasingly negative MU_e, and hence a greater propensity to avoid energy expenditure. The minimum possible e while bicycling (at $p = 0$) is $e = \delta_0 = 0.035m \approx 2.6$ kcal/min.

The MRS_{et} estimates in Fig. 1 differ for the two data sets, ranging from 0.1 to 0.5. Subjects differ in their estimated disutility from effort, but except for subject B the estimated MRS_{et}

Table 2 Sensitivity of equilibrium speed to variation in parameter values

Parameter	Central value	Variation	Resulting range of v^* (m/s)
δ_1	0.058	± 0.005	4.83–5.06
$m + m_b$	95	± 10	4.92–4.96
$A_F * C_D$	0.75	± 0.1	4.80–5.11
C_R	0.006	± 0.001	4.91–4.97
G	0%	± 1%	4.64-5.27
MRS_{et}	0.3	± 0.1	4.57–5.51

increases with e. Moreover, MRS_{et} appears to remain well above zero at all values of e. However, as described above, excluding control effects leads to overestimates of the true MRS_{et} at high speeds (i.e., low values of e) and underestimates at low speeds (i.e., high values of e). Thus, the actual MRS_{et} curves may be steeper than those shown in Fig. 1.

Sensitivity of the equilibrium speed

Table 2 shows how the equilibrium speed v^* given in Eq. (8) varies with changes in the traveler and roadway parameters. With the central parameter values, v^* is 4.94 m/s, and the grade constraint for application of Eq. (8) is $G > -2.3\%$ Eq. (9). Equilibrium speed is a decreasing function of each parameter listed in Table 2. The parameter variations considered are not directly comparable, but it appears that road grade (G) and MRS_{et} have the largest proportional effects on v^*, whereas bicycle and rider mass ($m + m_b$) and coefficient of rolling resistance (C_R) have the smallest effects. This sensitivity analysis is illustrative only since it is limited to variations in individual parameter values and relies on the MRS_{et} estimates shown in Fig. 1. A systematic analysis based on more extensive field data is clearly called for.

Application to electric bicycles

In this section the speed choice model is applied to electric bicycles for validation and to demonstrate how it can be used to predict active travel speeds. Attention is focused on pedal-assist electric bicycles which typically deliver power assistance that is proportional to the rider's own power input. (By contrast, "scooter-style" electric bicycles allow riders to control power delivery with a throttle.) Let a denote the power assistance level from the electric motor, expressed as a percentage of the rider's power input, p. Power assist levels can range from $a = 25\%$ to 250% (NYCeWheels; Prindle 2015).

Given a power assistance level a, the (non-zero) energy expenditure on a pedal-assist electric bicycle becomes

$$e = \delta_0 + \delta_1 \frac{1}{1+a} p = \delta_0 + \frac{\delta_1}{1+a}\left(\mu_1 v + \mu_3 v^3\right), \tag{10}$$

which leads to $\frac{de}{dv} = \frac{\delta_1}{1+a}\left(\mu_1 + 3\mu_3 v^2\right)$. With $a = 0$, this expression reduces to the previous formula for a conventional bicycle. Given a value of MRS_{et}, cruising speed v^* on an electric bicycle is

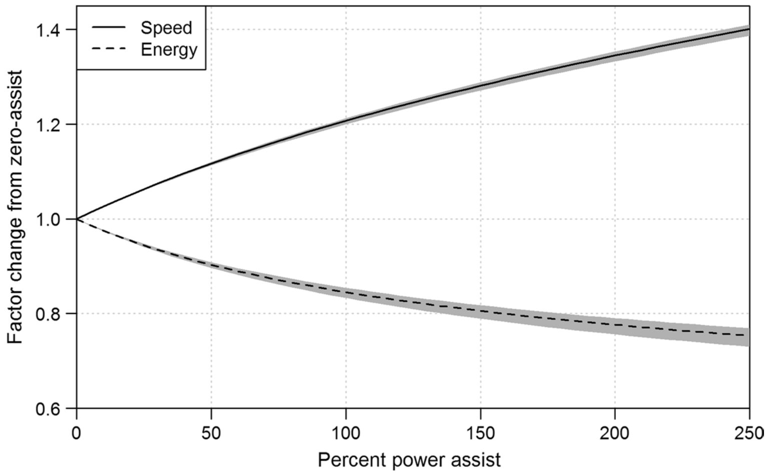

Fig. 2 Modeled speed and energy expenditure as a function of electric power assistance. The shaded area covers a range for MRS_{et} of 0.1–0.5

$$v^* = \sqrt{\frac{1}{6\mu_3}\left(\sqrt{\mu_1^2 + \frac{(1+a)200\mu_3}{\delta_1 MRS_{et}}} - \mu_1\right)} \tag{11}$$

with grade constraint $G > \frac{-1}{(m+m_b)g}\sqrt{\frac{(1+a)\mu_3}{0.12\delta_1 MRS_{et}}} - C_R$.

Figure 2 shows the modeled equilibrium speed (Eq. 11) and corresponding energy expenditure (Eq. 10) with increasing a for level riding ($G = 0$), using the Parkin parameters above and MRS_{et} of 0.3 (lines) and 0.1–0.5 (shaded areas). The speed choice model indicates that riders will use some of the benefit of power assistance to decrease their energy expenditure, and some of it to decrease their time costs by increasing speed. Note that this figure neglects the effects of control (which could suppress large speed increases at high power-assist levels), and possible relationships between a and the bicycle resistance parameters (which could arise from heavier bicycles with larger wheels, for example).

The narrow shaded area in Fig. 2 indicates that, despite the sensitivity of v^* to MRS_{et} shown in Table 2, the effect of a on v^* is not very sensitive to the value of MRS_{et}. Figure 3 gives the modeled relationship between v^* and a for three values of MRS_{et} using the same parameters as Fig. 2. Despite the wide range of v^* that can result from heterogeneity in MRS_{et}, the slopes of the lines in Fig. 3 are similar, which explains the narrow range of results in Fig. 2 (presented as factor changes with a fixed MRS_{et}). The other parameters in Table 2 and Eq. (11) have similarly small effects on the relationship between v^* and a, as long as they are independent of a—which might not be true for factors such as bicycle mass. Even if electric bicycles with $a > 0$ are 20 kg heavier, the factor changes in speed and energy from conventional bicycles ($a = 0$) are within about 2% of the values shown in Fig. 2.

In a recent study of five bicyclists in Lisbon, Portugal, average travel speeds were 8-26% faster on electric bicycles than conventional bicycles (Baptista et al. 2015). The electric battery supplied 33–56% of total energy use for these riders, which translates to a values

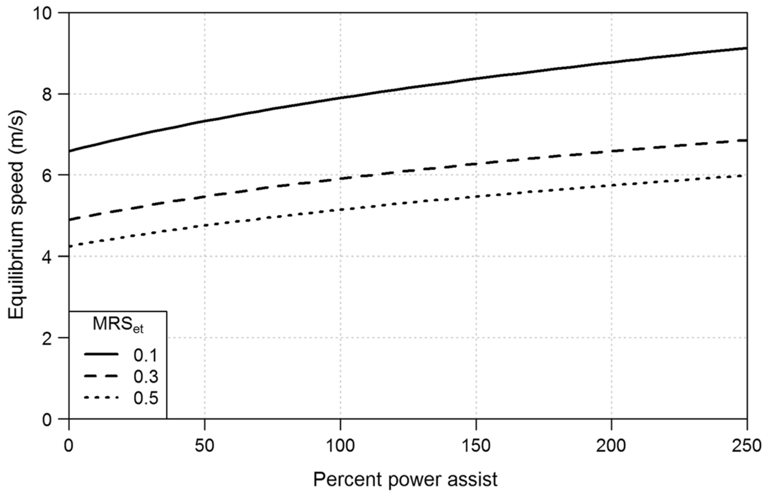

Fig. 3 Speed adaptation to power assistance at different values of MRS_{et}

of 49 to 127%. At these a values in Fig. 2, speed is 12–25% higher than at $a = 0$, which is remarkably similar to the speed increases observed in the study. Two other studies observed speed increases for electric versus conventional bicycles around 14% (Cherry and Cervero 2007) and 27% (Langford et al. 2015). In Fig. 2 these speed increases occur at $a = 60\%$ and $a = 140\%$, both of which are reasonable power assist levels. The model predicts general speed adaptation to power assistance well, despite neglecting control effects and modeling cruising speed on level ground rather than overall average speed.

In a broader analysis it could be of interest to consider the individuals most likely to transition to electric bicycles: presumably travelers with greater disutility of energy expenditure, and hence higher MRS_{et} and lower equilibrium speeds, *ceteris paribus*. If predominantly high-MRS_{et} travelers transition to electric bicycles, and as a result make higher speed choices (Fig. 3), it could lead to a narrower distribution of overall (conventional and electric) bicycle speeds. For example, all else equal, a rider with $a = 80\%$ power-assist and $MRS_{et} = 0.5$ is expected to have about the same equilibrium speed as a rider on a conventional bicycle, $a = 0$, and $MRS_{et} = 0.3$. This homogenizing influence could be enhanced by control effects that moderate speed increases. Alternatively, the range of speeds might increase if less risk-averse travelers (i.e., with less negative values of $MU_c \frac{dc}{dv}$ at high speeds) are more likely to adopt electric bicycles.

Discussion

The bicycle speed choice model presented in this paper is a tractable first step, but rests on a number of simplifying assumptions. First, it relies on a rational, utility maximization decision process that might not describe how active travellers make speed choices: a concern that arises with most types of rational travel behavior models (Heinen et al. 2010). In practice, speed choice may be largely habitual. Speed choice may also be influenced by social norms or through speed adaptation to other nearby modes with either higher (autos) or lower (pedestrians) speeds. Other bicyclists nearby could influence speed preference or choice by impeding movement, decreasing wind resistance, or changing the decision context to include relative speed as a source of utility (in cooperative or competitive groups) (Bernardi et al. 2015; Hatfield and Prabhakharan 2016). Bounded rationality arising from uncertainty may also be a factor as it is unlikely that bicyclists fully understand the long-term effects on their utility and well-being of a given level of energy expenditure.

A second major limitation of the model is that control effects are addressed only loosely. The fact that bicyclists brake on steep descents demonstrates that control effects cannot be ignored, at least in region III. Control effects in region I could similarly become relevant on a steep ascent. As explained, neglect of control effects leads to mis-estimation of the marginal rate of substitution between travel time and energy expenditure (MRS_{et}). For a given individual, MRS_{et} likely increases more steeply with effort than shown in Fig. 1. Across individuals, some of the heterogeneity in MRS_{et} may be due to interpersonal differences in control functions. For example, a risk-averse traveler might be more concerned about loss of control with increasing speed, leading to overestimation of MRS_{et} relative to other travelers. Better understanding of control utility functions would improve speed modeling, and help explain speed and risk-taking behavior by connecting speed choice to attitudes and behaviors such as helmet use and traffic violations (Bai et al. 2013; Fyhri et al. 2012).

A third limitation of the model is that it considers only steady-state conditions in which the bicyclist is cruising at a constant speed. This ignores not only changes in speed induced by variations in grade, wind speed, wind direction and so on, but also stop events due to intersections or traffic congestion. As stated in the introduction, route choice models have been estimated with a composite disutility for major road intersections and crossings (Broach et al. 2012; Sener et al. 2009), combining the effects of time and energy (and possibly crash risk) factors. The model in this paper can be extended to include deceleration/acceleration events, which would enable decomposition of the sources of disutility for bicyclist stops and provide insights into how stop costs are influenced by bicycle technology and personal characteristics.

Preferred speed is an important aspect of cycling behavior to understand regardless of the proportion of time that cyclists can travel at their preferred speeds. When external constraints restrict cyclist speeds, deceleration/acceleration dynamics depend on the target speed, and preferred speed is a common parameter for microsimulation models (Ma and Luo 2016; Twaddle et al. 2014; Twaddle and Grigoropoulos 2016). Understanding speed preferences also enables estimation of disutility from lower speeds when bicyclists are impeded by other road users, which could be used to model bicycle route choices affected by congestion and to evaluate the benefits of segregated and high-capacity bicycle facilities.

Bicyclists care about other factors besides time, energy, and control/safety (Heinen et al. 2010). General effects of bicycling on traveler utility that are independent of speed are not required to model speed choice. These effects likely include some safety concerns, some effects of exercise, and myriad reasons for avoiding auto travel. Unpleasant weather, a major concern for bicyclists, could influence speed choice through a greater disutility of travel time, which would result in a lower MRS_{et} and higher equilibrium speed (reflecting a desire to get off the road and out of inclement weather). At night, both lack of adequate street lighting and excessive light from oncoming vehicle headlights (as well as bicycle halogen lights) may induce bicyclists to slow down. The same is true of rough pavement and slippery conditions. Perspiration is another known impediment to bicycle commuting that probably increases disutility from effort, leading to a higher MRS_{et} and lower equilibrium speed (to avoid excessive sweat). Both inter-personal and intra-personal variations in MRS_{et} should be explored along with covariates (trip purpose and length, weather, etc.) in a broad population of travelers to help illuminate these and other influences on bicycling behavior.

To calibrate the model for a broader population of travelers, an important next step is to examine the joint distribution of the physical and physiological parameters (A_F, C_D, C_R, m, m_b, and δ_1) for real-world urban bicyclists. Correlations are likely to exist among these parameters as well as between these parameters and other factors such as age. In addition, applying the speed choice model to future bicycle fleets requires a projection of these parameters, which might evolve appreciably over time with the proliferation of electric bicycles, for example. New power meter technologies can be used to generate data for validation or empirical estimation of the relationship between power and speed (Eq. 5). In addition to these parameter values, model calibration requires a method to determine representative free-flow cruising speeds, by extraction from GPS data or possibly some other experimental method (Ma and Luo 2016).

Developing active travel speed choice models can lead to more sensitive and behaviorally-grounded route choice, mode choice, operations, and microsimulation models that account for the influences of bicycle and traveller attributes, road grade, wind, and other factors. For example, behavioral speed models could explain how electric bicycles influence traffic flow on bicycle facilities (Jin et al. 2015). This line of research is also working toward the development of route and mode choice models that separate time and energy effects, and are thus more sensitive to the growth in use of electric bicycles and other human/electric hybrid vehicles. Estimation of time/energy trade-offs such as MRS_{et} will allow valuation of energy expenditure in full social cost accounting (similar to travel time valuation). Variation in MRS_{et} could explain some of the observed heterogeneity in route and mode choices. To progress toward these goals, future work should explore methods for quantifying control utility functions, expand the model to include speed dynamics and other factors such as surface roughness, examine normalization effects of bicyclists in groups and bicyclists on different types of facilities (i.e., arterials with high-speed auto traffic vs. paths with low-speed pedestrian traffic), incorporate technical speed-flow and speed-density relationships such as (Jiang et al. 2016) to model speed choice under both unconstrained and constrained flow conditions, and develop a similar speed choice model for pedestrians.

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Alexander Bigazzi is an assistant professor at the University of British Columbia, with a joint appointment in the Department of Civil Engineering and the School of Community and Regional Planning. He received his Ph.D. from Portland State University in 2014, investigating urban cyclists' uptake of traffic-related air pollution. His primary research areas include transportation emissions and air quality, active travel behavior, traffic management and modeling, and the effects of urban transportation systems on health.

Robin Lindsey holds the CN Chair in Transportation and International Logistics at the Sauder School of Business, University of British Columbia. His research interests include traffic congestion, road pricing, financing transportation infrastructure, public transportation, and advanced traveler information systems. He has published articles in Transportation Research A, B, C, Transportation Science, American Economic Review, European Economic Review, International Economic Review, Journal of Public Economics, Journal of Urban Economics, Rand Journal of Economics, and other transportation and economics journals. Lindsey is past president of the International Transportation Economics Association, an Associate Editor of Transportation Research Part B and Transportmetrica, and a member of the editorial boards of Economics of Transportation, International Journal of Sustainable Transportation, Journal of Urban Economics, and Transport Policy.